

## Trigonometrical Identities

### Introduction

Very often it is necessary to rewrite expressions involving sines, cosines and tangents in alternative forms. To do this we use formulas known as **trigonometric identities**. A number of commonly used identities are listed here:

### 1. The identities

$$\tan A = \frac{\sin A}{\cos A} \quad \sec A = \frac{1}{\cos A} \quad \operatorname{cosec} A = \frac{1}{\sin A} \quad \cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A, \quad \tan^2 A + 1 = \sec^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}, \quad \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$$

$$\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\cos A - \cos B = 2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{B-A}{2} \right)$$

---

Note:  $\sin^2 A$  is the notation used for  $(\sin A)^2$ . Similarly  $\cos^2 A$  means  $(\cos A)^2$  and so on.