

## Graphs of the trig functions

### Introduction

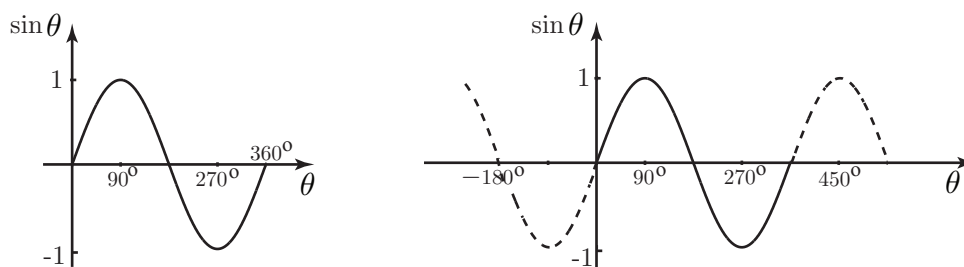
The trigonometric functions play a very important role in engineering mathematics. Familiarity with the graphs of these functions is essential. Graphs of the trigonometric functions sine, cosine and tangent, together with some tabulated values are shown here for reference.

### 1. The sine function

Using a scientific calculator a table of values of  $\sin \theta$  can be drawn up as  $\theta$  varies from  $0$  to  $360^\circ$ .

|               |             |             |             |             |             |             |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\theta$      | $0^\circ$   | $30^\circ$  | $60^\circ$  | $90^\circ$  | $120^\circ$ | $150^\circ$ |
| $\sin \theta$ | 0           | 0.5000      | 0.8660      | 1           | 0.8660      | 0.5000      |
| $\theta$      | $180^\circ$ | $210^\circ$ | $240^\circ$ | $270^\circ$ | $300^\circ$ | $360^\circ$ |
| $\sin \theta$ | 0           | -0.5000     | -0.8660     | -1          | -0.8660     | 0           |

Using the table, a graph of the function  $y = \sin \theta$  can be plotted and is shown below on the left.



If further values, outside the range  $0^\circ$  to  $360^\circ$ , are calculated we find that the wavy pattern repeats itself as shown on the right. We say that the sine function is **periodic** with period  $360^\circ$ . Some values are particularly important and should be remembered:

$$\sin 0^\circ = 0, \quad \sin 90^\circ = 1, \quad \sin 180^\circ = 0, \quad \sin 270^\circ = -1$$

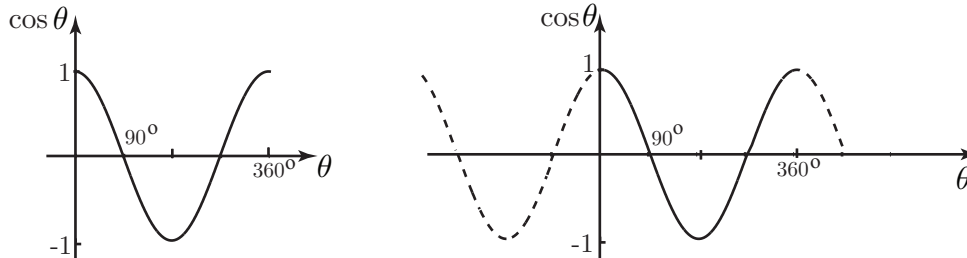
The maximum value of  $\sin \theta$  is 1, and the minimum value is  $-1$ .

### 2. The cosine function

Using a scientific calculator a table of values of  $\cos \theta$  can be drawn up as  $\theta$  varies from  $0$  to  $360^\circ$ .

|               |             |             |             |             |             |             |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\theta$      | $0^\circ$   | $30^\circ$  | $60^\circ$  | $90^\circ$  | $120^\circ$ | $150^\circ$ |
| $\cos \theta$ | 1           | 0.8660      | 0.5000      | 0           | -0.5000     | -0.8660     |
| $\theta$      | $180^\circ$ | $210^\circ$ | $240^\circ$ | $270^\circ$ | $300^\circ$ | $360^\circ$ |
| $\cos \theta$ | -1          | -0.8660     | -0.5000     | 0           | 0.5000      | 1           |

Using the table, a graph of the function  $y = \cos \theta$  can be plotted as shown on the left.



If further values are calculated outside the range  $0 \leq \theta \leq 360^\circ$  we find that the wavy pattern repeats itself as shown on the right. We say that the cosine function is periodic with period  $360^\circ$ . Some values are particularly important and should be remembered:

$$\cos 0^\circ = 1, \quad \cos 90^\circ = 0, \quad \cos 180^\circ = -1, \quad \cos 270^\circ = 0$$

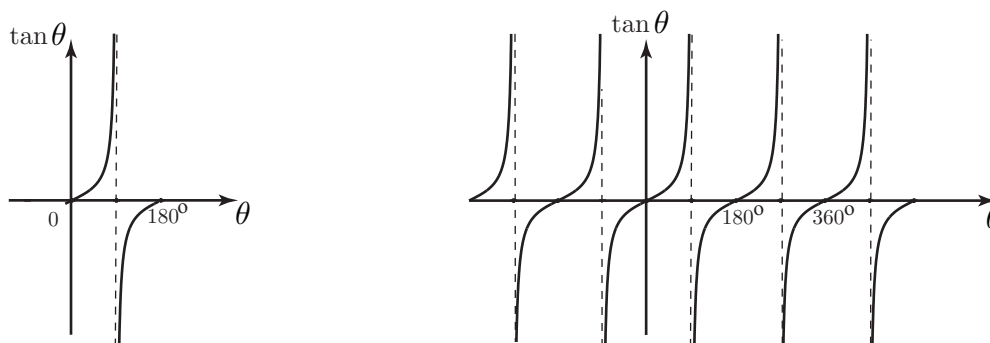
The maximum value of  $\cos \theta$  is 1, and the minimum value is  $-1$ .

### 3. The tangent function

Using a scientific calculator a table of values of  $\tan \theta$  can be drawn up as  $\theta$  varies from  $0$  to  $180^\circ$  although when  $\theta = 90^\circ$  you will find that this function is not defined.

|               |     |            |            |             |             |
|---------------|-----|------------|------------|-------------|-------------|
| $\theta$      | $0$ | $45^\circ$ | $90^\circ$ | $135^\circ$ | $180^\circ$ |
| $\tan \theta$ | 0   | 1          | $\infty$   | -1          | 0           |

Using the table, a graph of the function  $y = \tan \theta$  can be plotted and is shown below on the left.



If further values are calculated outside the range  $0 \leq \theta \leq 180^\circ$  we find that the pattern repeats itself as shown on the right. We say that the tangent function is periodic with period  $180^\circ$ .

Some values are particularly important and should be remembered:

$$\tan 0^\circ = 0, \quad \tan 45^\circ = 1$$

There is no maximum value of  $\tan \theta$  because it increases without bound. There is no minimum value. However there are certain values where  $\tan \theta$  is not defined, including  $-90^\circ$ ,  $90^\circ$ ,  $270^\circ$  and so on. Here the graph shoots off to infinity.