

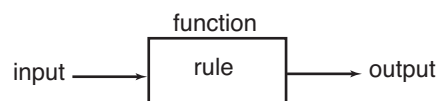
What is a function ?

Introduction

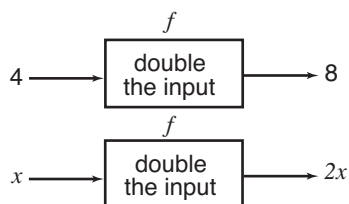
A quantity whose value can change is known as a **variable**. **Functions** are used to describe the rules which define the ways in which such a change can occur. The purpose of this leaflet is to explain functions and their notation.

1. The function rule

A function is a rule which operates on an **input** and produces an **output**. This can be illustrated using a **block diagram** such as that shown below. We can think of the function as a mathematical machine which processes the input, using a given rule, in order to produce an output. We often write the rule inside the box.



In order for a rule to be a function it must produce only a single output for any given input. The function with the rule 'double the input' is shown below.



Note that with an input of 4 the function would produce an output of 8. With a more general input, x say, the output will be $2x$. It is usual to assign a letter or other symbol to a function in order to label it. The doubling function pictured in the example above has been given the symbol f .

A function is a rule which operates on an input and produces a single output from that input.

For the doubling function it is common to use the notation

$$f(x) = 2x$$

This indicates that with an input x , the function, f , produces an output of $2x$. The input to the function is placed in the brackets after the function label ' f '. $f(x)$ is read as ' f is a function of x ', or simply ' f of x ', meaning that the output from the function depends upon the value of the input x .

Example

State the rule of each of the following functions:

a) $f(x) = 7x + 9$, b) $h(t) = t^3 + 2$, c) $p(x) = x^3 + 2$.

Solution

a) The rule for f is ‘multiply the input by 7 and then add 9’.

b) The rule for h is ‘cube the input and add 2’.

c) The rule for p is ‘cube the input and add 2’.

Note from parts b) and c) that it is the rule that is important when describing a function and not the letters being used. Both $h(t)$ and $p(x)$ instruct us to ‘cube the input and add 2’.

The input to a function is called its **argument**. We can obtain the output from a function if we are given its argument. For example, given the function $f(x) = 3x + 2$ we may require the value of the output when the argument is 5. We write this as $f(5)$. Here, $f(5) = 3 \times 5 + 2 = 17$.

Example

Given the function $f(x) = 4x + 3$ find a) $f(-1)$, b) $f(6)$

Solution

a) Here the argument is -1 . We find $f(-1) = 4 \times (-1) + 3 = -1$.

b) $f(6) = 4(6) + 3 = 27$.

Sometimes the argument will be an algebraic expression, as in the following example.

Example

Given the function $y(x) = 5x - 3$ find

a) $y(t)$, b) $y(7t)$, c) $y(z + 2)$.

Solution

The function rule is multiply the input by 5, and subtract 3. We can apply this rule whatever the argument.

a) To find $y(t)$ multiply the argument, t , by 5 and subtract 3 to give $y(t) = 5t - 3$.

b) Now the argument is $7t$. So $y(7t) = 5(7t) - 3 = 35t - 3$.

c) In this case the argument is $z + 2$. We find $y(z + 2) = 5(z + 2) - 3 = 5z + 10 - 3 = 5z + 7$.

Exercises

1. Write down a function which can be used to describe the following rules:

a) ‘cube the input and divide the result by 2’, b) ‘divide the input by 5 and then add 7’

2. Given the function $f(x) = 7x - 3$ find a) $f(3)$, b) $f(6)$, c) $f(-2)$.

3. If $g(t) = t^2$ write down expressions for a) $g(x)$, b) $g(3t)$, c) $g(x + 4)$.

Answers

1. a) $f(x) = \frac{x^3}{2}$, b) $f(x) = \frac{x}{5} + 7$. 2. a) 18, b) 39, c) -17

3. a) $g(x) = x^2$, b) $g(3t) = (3t)^2 = 9t^2$, c) $g(x + 4) = (x + 4)^2 = x^2 + 8x + 16$.