

Bases other than 10 and e

Introduction

Occasionally you may need to find logarithms to bases other than 10 and e. For example, logarithms to the base 2 are used in communications engineering and information technology. Your calculator can still be used but we need to apply a formula for changing the base. This leaflet gives this formula and shows how to use it.

1. A formula for change of base

Suppose we want to calculate a logarithm to base 2. The formula states

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2}$$

So we can calculate base 2 logarithms using base 10 logarithms obtained using a calculator. For example

$$\log_2 36 = \frac{\log_{10} 36}{\log_{10} 2} = \frac{1.5563}{0.3010} = 5.1704$$

Check this for yourself.

More generally, for any bases a and b ,

$$\log_a x = \frac{\log_b x}{\log_b a}$$

In particular, by choosing $b = 10$ we find

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

Use this formula to check that $\log_{20} 100 = 1.5372$.

Exercises

1. Find a) $\log_2 15$, b) $\log_2 56.25$, c) $\log_3 16$.

Answers

1. a) 3.907 (3dp), b) 5.814 (3dp), c) 2.524 (3dp).