

# Graphical solution of inequalities

## Introduction

Graphs can be used to solve inequalities. This leaflet illustrates how.

## 1. Solving inequalities

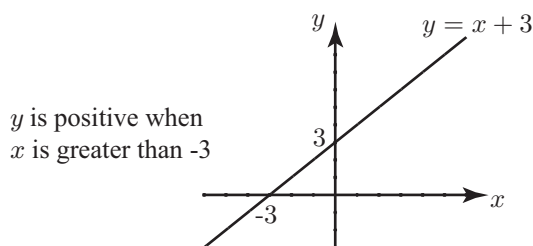
We start with a very simple example which could be solved very easily using an algebraic method.

### Example

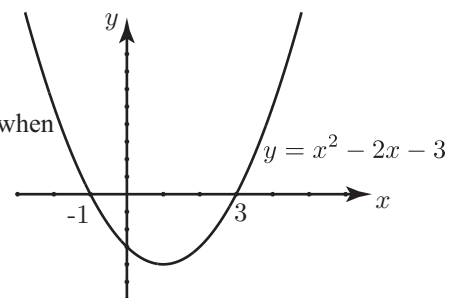
Solve the inequality  $x + 3 > 0$ .

### Solution

We seek values of  $x$  which make  $x + 3$  positive. There are many such values, e.g. try  $x = 7$  or  $x = -2$ . To find all values first let  $y = x + 3$ . Then the graph of  $y = x + 3$  is sketched as shown below. From the graph we see that the  $y$  coordinate of any point on the line is positive whenever  $x$  has a value greater than  $-3$ . That is,  $y > 0$  when  $x > -3$ . But  $y = x + 3$ , so we can conclude that  $x + 3$  will be positive when  $x > -3$ . We have used the graph to solve the inequality.



$y$  is positive when  $x$  is less than -1 and when  $x$  is greater than 3



### Example

Solve the inequality  $x^2 - 2x - 3 > 0$ .

### Solution

We seek values of  $x$  which make  $x^2 - 2x - 3$  positive. We can find these by sketching a graph of  $y = x^2 - 2x - 3$ . To help with the sketch, note that by factorising we can write  $y$  as  $(x+1)(x-3)$ . The graph will cross the horizontal axis when  $x = -1$  and when  $x = 3$ . The graph is shown

above on the right. From the graph note that the  $y$  coordinate of a point on the graph is positive when either  $x$  is greater than 3 or when  $x$  is less than  $-1$ . That is,  $y > 0$  when  $x > 3$  or  $x < -1$  and so:

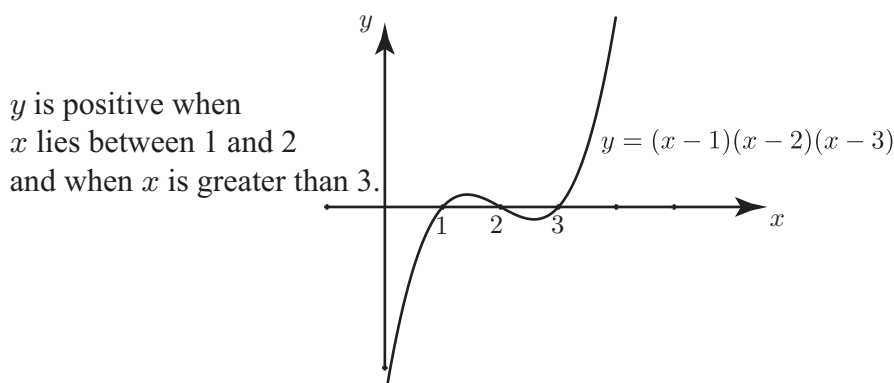
$$x^2 - 2x - 3 > 0 \quad \text{when} \quad x > 3 \quad \text{or} \quad x < -1$$

### Example

Solve the inequality  $(x - 1)(x - 2)(x - 3) > 0$ .

### Solution

We consider the graph of  $y = (x - 1)(x - 2)(x - 3)$  which is shown below. It is evident from the graph that  $y$  is positive when  $x$  lies between 1 and 2 and also when  $x$  is greater than 3. The solution of the inequality is therefore  $1 < x < 2$  and  $x > 3$ .



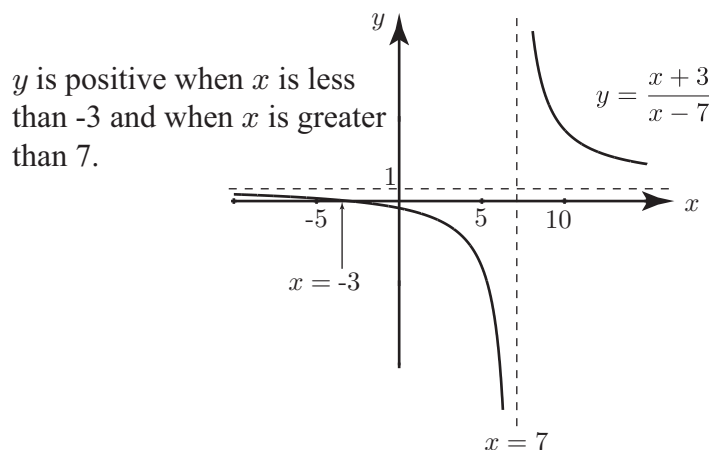
### Example

For what values of  $x$  is  $\frac{x+3}{x-7}$  positive ?

### Solution

The graph of  $y = \frac{x+3}{x-7}$  is shown below. We can see that the  $y$  coordinate of a point on the graph is positive when  $x < -3$  or when  $x > 7$ .

$$\frac{x + 3}{x - 7} > 0 \quad \text{when} \quad x < -3 \quad \text{or when} \quad x > 7$$



For drawing graphs like this one a graphical calculator is useful.